

Class XII : Maths
Chapter 7 : INTEGRALS

Questions and Solutions | Exercise 7.3 - NCERT Books

Question 1:

$$\sin^2(2x+5)$$

Answer

$$\sin^2(2x+5) = \frac{1 - \cos 2(2x+5)}{2} = \frac{1 - \cos(4x+10)}{2}$$

$$\begin{aligned} \Rightarrow \int \sin^2(2x+5) dx &= \int \frac{1 - \cos(4x+10)}{2} dx \\ &= \frac{1}{2} \int 1 dx - \frac{1}{2} \int \cos(4x+10) dx \\ &= \frac{1}{2}x - \frac{1}{2} \left(\frac{\sin(4x+10)}{4} \right) + C \\ &= \frac{1}{2}x - \frac{1}{8} \sin(4x+10) + C \end{aligned}$$

Question 2:

$$\sin 3x \cos 4x$$

Answer

It is known that, $\sin A \cos B = \frac{1}{2} \{ \sin(A+B) + \sin(A-B) \}$

$$\begin{aligned} \therefore \int \sin 3x \cos 4x dx &= \frac{1}{2} \int \{ \sin(3x+4x) + \sin(3x-4x) \} dx \\ &= \frac{1}{2} \int \{ \sin 7x + \sin(-x) \} dx \\ &= \frac{1}{2} \int \{ \sin 7x - \sin x \} dx \\ &= \frac{1}{2} \int \sin 7x dx - \frac{1}{2} \int \sin x dx \\ &= \frac{1}{2} \left(\frac{-\cos 7x}{7} \right) - \frac{1}{2} (-\cos x) + C \\ &= \frac{-\cos 7x}{14} + \frac{\cos x}{2} + C \end{aligned}$$

Question 3:

$$\cos 2x \cos 4x \cos 6x$$

Answer

$$\text{It is known that, } \cos A \cos B = \frac{1}{2} \{ \cos(A+B) + \cos(A-B) \}$$

$$\begin{aligned} \therefore \int \cos 2x (\cos 4x \cos 6x) dx &= \int \cos 2x \left[\frac{1}{2} \{ \cos(4x+6x) + \cos(4x-6x) \} \right] dx \\ &= \frac{1}{2} \int \{ \cos 2x \cos 10x + \cos 2x \cos(-2x) \} dx \\ &= \frac{1}{2} \int \{ \cos 2x \cos 10x + \cos^2 2x \} dx \\ &= \frac{1}{2} \int \left[\left\{ \frac{1}{2} \cos(2x+10x) + \cos(2x-10x) \right\} + \left(\frac{1+\cos 4x}{2} \right) \right] dx \\ &= \frac{1}{4} \int (\cos 12x + \cos 8x + 1 + \cos 4x) dx \\ &= \frac{1}{4} \left[\frac{\sin 12x}{12} + \frac{\sin 8x}{8} + x + \frac{\sin 4x}{4} \right] + C \end{aligned}$$

Question 4:

$$\sin^3 (2x + 1)$$

Answer

$$\text{Let } I = \int \sin^3 (2x+1)$$

$$\begin{aligned} \Rightarrow \int \sin^3 (2x+1) dx &= \int \sin^2 (2x+1) \cdot \sin(2x+1) dx \\ &= \int (1 - \cos^2 (2x+1)) \sin(2x+1) dx \end{aligned}$$

$$\text{Let } \cos(2x+1) = t$$

$$\Rightarrow -2 \sin(2x+1) dx = dt$$

$$\Rightarrow \sin(2x+1) dx = \frac{-dt}{2}$$

$$\begin{aligned}
 \Rightarrow I &= \frac{-1}{2} \int (1-t^2) dt \\
 &= \frac{-1}{2} \left\{ t - \frac{t^3}{3} \right\} \\
 &= \frac{-1}{2} \left\{ \cos(2x+1) - \frac{\cos^3(2x+1)}{3} \right\} \\
 &= \frac{-\cos(2x+1)}{2} + \frac{\cos^3(2x+1)}{6} + C
 \end{aligned}$$

Question 5:

$$\sin^3 x \cos^3 x$$

Answer

$$\begin{aligned}
 \text{Let } I &= \int \sin^3 x \cos^3 x \cdot dx \\
 &= \int \cos^3 x \cdot \sin^2 x \cdot \sin x \cdot dx \\
 &= \int \cos^3 x (1 - \cos^2 x) \sin x \cdot dx
 \end{aligned}$$

$$\text{Let } \cos x = t$$

$$\Rightarrow -\sin x \cdot dx = dt$$

$$\begin{aligned}
 \Rightarrow I &= -\int t^3 (1-t^2) dt \\
 &= -\int (t^3 - t^5) dt \\
 &= -\left\{ \frac{t^4}{4} - \frac{t^6}{6} \right\} + C \\
 &= -\left\{ \frac{\cos^4 x}{4} - \frac{\cos^6 x}{6} \right\} + C \\
 &= \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + C
 \end{aligned}$$

Question 6:

$$\sin x \sin 2x \sin 3x$$

Answer

It is known that, $\sin A \sin B = \frac{1}{2} \{ \cos(A - B) - \cos(A + B) \}$

$$\begin{aligned} \therefore \int \sin x \sin 2x \sin 3x \, dx &= \int \left[\sin x \cdot \frac{1}{2} \{ \cos(2x - 3x) - \cos(2x + 3x) \} \right] dx \\ &= \frac{1}{2} \int (\sin x \cos(-x) - \sin x \cos 5x) \, dx \\ &= \frac{1}{2} \int (\sin x \cos x - \sin x \cos 5x) \, dx \\ &= \frac{1}{2} \int \frac{\sin 2x}{2} \, dx - \frac{1}{2} \int \sin x \cos 5x \, dx \\ &= \frac{1}{4} \left[\frac{-\cos 2x}{2} \right] - \frac{1}{2} \int \left\{ \frac{1}{2} \sin(x + 5x) + \sin(x - 5x) \right\} dx \\ &= \frac{-\cos 2x}{8} - \frac{1}{4} \int (\sin 6x + \sin(-4x)) \, dx \\ &= \frac{-\cos 2x}{8} - \frac{1}{4} \left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{4} \right] + C \\ &= \frac{-\cos 2x}{8} - \frac{1}{8} \left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{2} \right] + C \\ &= \frac{1}{8} \left[\frac{\cos 6x}{3} - \frac{\cos 4x}{2} - \cos 2x \right] + C \end{aligned}$$

Question 7:

$\sin 4x \sin 8x$

Answer

It is known that, $\sin A \sin B = \frac{1}{2} \cos(A - B) - \cos(A + B)$

$$\begin{aligned}
 \therefore \int \sin 4x \sin 8x \, dx &= \int \left\{ \frac{1}{2} \cos(4x - 8x) - \cos(4x + 8x) \right\} dx \\
 &= \frac{1}{2} \int (\cos(-4x) - \cos 12x) \, dx \\
 &= \frac{1}{2} \int (\cos 4x - \cos 12x) \, dx \\
 &= \frac{1}{2} \left[\frac{\sin 4x}{4} - \frac{\sin 12x}{12} \right]
 \end{aligned}$$

Question 8:

$$\frac{1 - \cos x}{1 + \cos x}$$

Answer

$$\begin{aligned}
 \frac{1 - \cos x}{1 + \cos x} &= \frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \quad \left[2 \sin^2 \frac{x}{2} = 1 - \cos x \text{ and } 2 \cos^2 \frac{x}{2} = 1 + \cos x \right] \\
 &= \tan^2 \frac{x}{2} \\
 &= \left(\sec^2 \frac{x}{2} - 1 \right) \\
 \therefore \int \frac{1 - \cos x}{1 + \cos x} dx &= \int \left(\sec^2 \frac{x}{2} - 1 \right) dx \\
 &= \left[\frac{\tan \frac{x}{2}}{1} - x \right] + C \\
 &= 2 \tan \frac{x}{2} - x + C
 \end{aligned}$$

Question 9:

$$\frac{\cos x}{1 + \cos x}$$

Answer

$$\frac{\cos x}{1 + \cos x} = \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \quad \left[\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \text{ and } \cos x = 2 \cos^2 \frac{x}{2} - 1 \right]$$
$$= \frac{1}{2} \left[1 - \tan^2 \frac{x}{2} \right]$$

$$\therefore \int \frac{\cos x}{1 + \cos x} dx = \frac{1}{2} \int \left(1 - \tan^2 \frac{x}{2} \right) dx$$
$$= \frac{1}{2} \int \left(1 - \sec^2 \frac{x}{2} + 1 \right) dx$$
$$= \frac{1}{2} \int \left(2 - \sec^2 \frac{x}{2} \right) dx$$
$$= \frac{1}{2} \left[2x - \frac{\tan \frac{x}{2}}{\frac{1}{2}} \right] + C$$
$$= x - \tan \frac{x}{2} + C$$

Question 10:

$\sin^4 x$

Answer

$$\begin{aligned}\sin^4 x &= \sin^2 x \sin^2 x \\ &= \left(\frac{1 - \cos 2x}{2}\right) \left(\frac{1 - \cos 2x}{2}\right) \\ &= \frac{1}{4}(1 - \cos 2x)^2 \\ &= \frac{1}{4}[1 + \cos^2 2x - 2 \cos 2x] \\ &= \frac{1}{4}\left[1 + \left(\frac{1 + \cos 4x}{2}\right) - 2 \cos 2x\right] \\ &= \frac{1}{4}\left[1 + \frac{1}{2} + \frac{1}{2} \cos 4x - 2 \cos 2x\right] \\ &= \frac{1}{4}\left[\frac{3}{2} + \frac{1}{2} \cos 4x - 2 \cos 2x\right]\end{aligned}$$

$$\begin{aligned}\therefore \int \sin^4 x \, dx &= \frac{1}{4} \int \left[\frac{3}{2} + \frac{1}{2} \cos 4x - 2 \cos 2x\right] dx \\ &= \frac{1}{4} \left[\frac{3}{2}x + \frac{1}{2} \left(\frac{\sin 4x}{4}\right) - \frac{2 \sin 2x}{2}\right] + C \\ &= \frac{1}{8} \left[3x + \frac{\sin 4x}{4} - 2 \sin 2x\right] + C \\ &= \frac{3x}{8} - \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + C\end{aligned}$$

Question 11:

$\cos^4 2x$

Answer

$$\begin{aligned}
 \cos^4 2x &= (\cos^2 2x)^2 \\
 &= \left(\frac{1 + \cos 4x}{2}\right)^2 \\
 &= \frac{1}{4} [1 + \cos^2 4x + 2 \cos 4x] \\
 &= \frac{1}{4} \left[1 + \left(\frac{1 + \cos 8x}{2}\right) + 2 \cos 4x\right] \\
 &= \frac{1}{4} \left[1 + \frac{1}{2} + \frac{\cos 8x}{2} + 2 \cos 4x\right] \\
 &= \frac{1}{4} \left[\frac{3}{2} + \frac{\cos 8x}{2} + 2 \cos 4x\right] \\
 \therefore \int \cos^4 2x \, dx &= \int \left(\frac{3}{8} + \frac{\cos 8x}{8} + \frac{\cos 4x}{2}\right) dx \\
 &= \frac{3}{8}x + \frac{\sin 8x}{64} + \frac{\sin 4x}{8} + C
 \end{aligned}$$

Question 12:

$$\frac{\sin^2 x}{1 + \cos x}$$

Answer

$$\begin{aligned}
 \frac{\sin^2 x}{1 + \cos x} &= \frac{\left(2 \sin \frac{x}{2} \cos \frac{x}{2}\right)^2}{2 \cos^2 \frac{x}{2}} \quad \left[\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}; \cos x = 2 \cos^2 \frac{x}{2} - 1\right] \\
 &= \frac{4 \sin^2 \frac{x}{2} \cos^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \\
 &= 2 \sin^2 \frac{x}{2} \\
 &= 1 - \cos x \\
 \therefore \int \frac{\sin^2 x}{1 + \cos x} dx &= \int (1 - \cos x) dx \\
 &= x - \sin x + C
 \end{aligned}$$

Question 13:

$$\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$$

Answer

$$\begin{aligned} \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} &= \frac{-2 \sin \frac{2x+2\alpha}{2} \sin \frac{2x-2\alpha}{2}}{-2 \sin \frac{x+\alpha}{2} \sin \frac{x-\alpha}{2}} \quad \left[\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2} \right] \\ &= \frac{\sin(x+\alpha) \sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right) \sin\left(\frac{x-\alpha}{2}\right)} \\ &= \frac{\left[2 \sin\left(\frac{x+\alpha}{2}\right) \cos\left(\frac{x+\alpha}{2}\right) \right] \left[2 \sin\left(\frac{x-\alpha}{2}\right) \cos\left(\frac{x-\alpha}{2}\right) \right]}{\sin\left(\frac{x+\alpha}{2}\right) \sin\left(\frac{x-\alpha}{2}\right)} \\ &= 4 \cos\left(\frac{x+\alpha}{2}\right) \cos\left(\frac{x-\alpha}{2}\right) \\ &= 2 \left[\cos\left(\frac{x+\alpha}{2} + \frac{x-\alpha}{2}\right) + \cos\left(\frac{x+\alpha}{2} - \frac{x-\alpha}{2}\right) \right] \\ &= 2 [\cos(x) + \cos \alpha] \\ &= 2 \cos x + 2 \cos \alpha \\ \therefore \int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx &= \int 2 \cos x + 2 \cos \alpha \\ &= 2 [\sin x + x \cos \alpha] + C \end{aligned}$$

Question 14:

$$\frac{\cos x - \sin x}{1 + \sin 2x}$$

Answer

$$\frac{\cos x - \sin x}{1 + \sin 2x} = \frac{\cos x - \sin x}{(\sin^2 x + \cos^2 x) + 2 \sin x \cos x}$$

$$\left[\sin^2 x + \cos^2 x = 1; \sin 2x = 2 \sin x \cos x \right]$$

$$= \frac{\cos x - \sin x}{(\sin x + \cos x)^2}$$

Let $\sin x + \cos x = t$

$$\therefore (\cos x - \sin x) dx = dt$$

$$\Rightarrow \int \frac{\cos x - \sin x}{1 + \sin 2x} dx = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx$$

$$= \int \frac{dt}{t^2}$$

$$= \int t^{-2} dt$$

$$= -t^{-1} + C$$

$$= -\frac{1}{t} + C$$

$$= \frac{-1}{\sin x + \cos x} + C$$

Question 15:

$$\tan^3 2x \sec 2x$$

Answer

$$\begin{aligned}\tan^3 2x \sec 2x &= \tan^2 2x \tan 2x \sec 2x \\ &= (\sec^2 2x - 1) \tan 2x \sec 2x \\ &= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x \\ \therefore \int \tan^3 2x \sec 2x \, dx &= \int \sec^2 2x \tan 2x \sec 2x \, dx - \int \tan 2x \sec 2x \, dx \\ &= \int \sec^2 2x \tan 2x \sec 2x \, dx - \frac{\sec 2x}{2} + C\end{aligned}$$

Let $\sec 2x = t$

$$\therefore 2 \sec 2x \tan 2x \, dx = dt$$

$$\begin{aligned}\therefore \int \tan^3 2x \sec 2x \, dx &= \frac{1}{2} \int t^2 dt - \frac{\sec 2x}{2} + C \\ &= \frac{t^3}{6} - \frac{\sec 2x}{2} + C \\ &= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C\end{aligned}$$

Question 16:

$$\tan^4 x$$

Answer

$$\begin{aligned}\tan^4 x &= \tan^2 x \cdot \tan^2 x \\ &= (\sec^2 x - 1) \tan^2 x \\ &= \sec^2 x \tan^2 x - \tan^2 x \\ &= \sec^2 x \tan^2 x - (\sec^2 x - 1) \\ &= \sec^2 x \tan^2 x - \sec^2 x + 1\end{aligned}$$

$$\begin{aligned}\therefore \int \tan^4 x \, dx &= \int \sec^2 x \tan^2 x \, dx - \int \sec^2 x \, dx + \int 1 \cdot dx \\ &= \int \sec^2 x \tan^2 x \, dx - \tan x + x + C \quad \dots(1)\end{aligned}$$

Consider $\int \sec^2 x \tan^2 x \, dx$

$$\text{Let } \tan x = t \Rightarrow \sec^2 x \, dx = dt$$

$$\Rightarrow \int \sec^2 x \tan^2 x \, dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$$

From equation (1), we obtain

$$\int \tan^4 x \, dx = \frac{1}{3} \tan^3 x - \tan x + x + C$$

Question 17:

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$$

Answer

$$\begin{aligned} \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} &= \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x} \\ &= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x} \\ &= \tan x \sec x + \cot x \operatorname{cosec} x \end{aligned}$$

$$\begin{aligned} \therefore \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} \, dx &= \int (\tan x \sec x + \cot x \operatorname{cosec} x) \, dx \\ &= \sec x - \operatorname{cosec} x + C \end{aligned}$$

Question 18:

$$\frac{\cos 2x + 2 \sin^2 x}{\cos^2 x}$$

Answer

$$\begin{aligned} \frac{\cos 2x + 2 \sin^2 x}{\cos^2 x} &= \frac{\cos 2x + (1 - \cos 2x)}{\cos^2 x} \quad [\cos 2x = 1 - 2 \sin^2 x] \\ &= \frac{1}{\cos^2 x} \\ &= \sec^2 x \end{aligned}$$

$$\therefore \int \frac{\cos 2x + 2 \sin^2 x}{\cos^2 x} \, dx = \int \sec^2 x \, dx = \tan x + C$$

Question 19:

$$\frac{1}{\sin x \cos^3 x}$$

Answer

$$\begin{aligned}\frac{1}{\sin x \cos^3 x} &= \frac{\sin^2 x + \cos^2 x}{\sin x \cos^3 x} \\ &= \frac{\sin x}{\cos^3 x} + \frac{1}{\sin x \cos x} \\ &= \tan x \sec^2 x + \frac{1 \cos^2 x}{\sin x \cos x} \\ &= \tan x \sec^2 x + \frac{\sec^2 x}{\tan x}\end{aligned}$$

$$\therefore \int \frac{1}{\sin x \cos^3 x} dx = \int \tan x \sec^2 x dx + \int \frac{\sec^2 x}{\tan x} dx$$

$$\text{Let } \tan x = t \Rightarrow \sec^2 x dx = dt$$

$$\begin{aligned}\Rightarrow \int \frac{1}{\sin x \cos^3 x} dx &= \int t dt + \int \frac{1}{t} dt \\ &= \frac{t^2}{2} + \log|t| + C \\ &= \frac{1}{2} \tan^2 x + \log|\tan x| + C\end{aligned}$$

Question 20:

$$\frac{\cos 2x}{(\cos x + \sin x)^2}$$

Answer

$$\frac{\cos 2x}{(\cos x + \sin x)^2} = \frac{\cos 2x}{\cos^2 x + \sin^2 x + 2 \sin x \cos x} = \frac{\cos 2x}{1 + \sin 2x}$$

$$\therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \int \frac{\cos 2x}{1 + \sin 2x} dx$$

$$\text{Let } 1 + \sin 2x = t$$

$$\Rightarrow 2 \cos 2x dx = dt$$

$$\begin{aligned} \therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx &= \frac{1}{2} \int \frac{1}{t} dt \\ &= \frac{1}{2} \log |t| + C \\ &= \frac{1}{2} \log |1 + \sin 2x| + C \\ &= \frac{1}{2} \log |(\sin x + \cos x)^2| + C \\ &= \log |\sin x + \cos x| + C \end{aligned}$$

Question 21:

$$\sin^{-1}(\cos x)$$

Answer

$$\sin^{-1}(\cos x)$$

$$\text{Let } \cos x = t$$

$$\text{Then, } \sin x = \sqrt{1 - t^2}$$

$$\Rightarrow (-\sin x) dx = dt$$

$$dx = \frac{-dt}{\sin x}$$

$$dx = \frac{-dt}{\sqrt{1-t^2}}$$

$$\begin{aligned} \therefore \int \sin^{-1}(\cos x) dx &= \int \sin^{-1}t \left(\frac{-dt}{\sqrt{1-t^2}} \right) \\ &= - \int \frac{\sin^{-1}t}{\sqrt{1-t^2}} dt \end{aligned}$$

Let $\sin^{-1}t = u$

$$\Rightarrow \frac{1}{\sqrt{1-t^2}} dt = du$$

$$\begin{aligned} \therefore \int \sin^{-1}(\cos x) dx &= \int -u du \\ &= -\frac{u^2}{2} + C \\ &= -\frac{(\sin^{-1}t)^2}{2} + C \\ &= -\frac{[\sin^{-1}(\cos x)]^2}{2} + C \quad \dots(1) \end{aligned}$$

It is known that,

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

$$\therefore \sin^{-1}(\cos x) = \frac{\pi}{2} - \cos^{-1}(\cos x) = \left(\frac{\pi}{2} - x \right)$$

Substituting in equation (1), we obtain

$$\begin{aligned} \int \sin^{-1}(\cos x) dx &= -\frac{\left[\frac{\pi}{2} - x \right]^2}{2} + C \\ &= -\frac{1}{2} \left(\frac{\pi^2}{2} + x^2 - \pi x \right) + C \\ &= -\frac{\pi^2}{8} - \frac{x^2}{2} + \frac{1}{2} \pi x + C \\ &= \frac{\pi x}{2} - \frac{x^2}{2} + \left(C - \frac{\pi^2}{8} \right) \\ &= \frac{\pi x}{2} - \frac{x^2}{2} + C_1 \end{aligned}$$

Question 22:

$$\frac{1}{\cos(x-a)\cos(x-b)}$$

Answer

$$\begin{aligned} \frac{1}{\cos(x-a)\cos(x-b)} &= \frac{1}{\sin(a-b)} \left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)} \right] \\ &= \frac{1}{\sin(a-b)} \left[\frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)} \right] \\ &= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)} \right] \\ &= \frac{1}{\sin(a-b)} [\tan(x-b) - \tan(x-a)] \end{aligned}$$

$$\begin{aligned} \Rightarrow \int \frac{1}{\cos(x-a)\cos(x-b)} dx &= \frac{1}{\sin(a-b)} \int [\tan(x-b) - \tan(x-a)] dx \\ &= \frac{1}{\sin(a-b)} [-\log|\cos(x-b)| + \log|\cos(x-a)|] \\ &= \frac{1}{\sin(a-b)} \left[\log \left| \frac{\cos(x-a)}{\cos(x-b)} \right| \right] + C \end{aligned}$$

Question 23:

$$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx \text{ is equal to}$$

- A. $\tan x + \cot x + C$
- B. $\tan x + \operatorname{cosec} x + C$
- C. $-\tan x + \cot x + C$
- D. $\tan x + \sec x + C$

Answer

$$\begin{aligned} \int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx &= \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx \\ &= \int (\sec^2 x - \operatorname{cosec}^2 x) dx \\ &= \tan x + \cot x + C \end{aligned}$$

Hence, the correct Answer is A.

Question 24:

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$$

equals

- A. $-\cot(e^{x^x}) + C$
- B. $\tan(xe^x) + C$
- C. $\tan(e^x) + C$
- D. $\cot(e^x) + C$

Answer

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$$

$$\text{Let } e^{x^x} = t$$

$$\Rightarrow (e^x \cdot x + e^x \cdot 1) dx = dt$$

$$e^x(x+1) dx = dt$$

$$\begin{aligned} \therefore \int \frac{e^x(1+x)}{\cos^2(e^x x)} dx &= \int \frac{dt}{\cos^2 t} \\ &= \int \sec^2 t \, dt \\ &= \tan t + C \\ &= \tan(e^x \cdot x) + C \end{aligned}$$

Hence, the correct Answer is B.